

Topics : Application of Derivatives, Solution of Triangle

Type of Questions		M.M., Min.
Comprehension (no negative marking) Q.1 to Q.3	(3 marks, 3 min.)	[9, 9]
Single choice Objective (no negative marking) Q. 4,5,6,7	(3 marks, 3 min.)	[12, 12]
Subjective Questions (no negative marking) Q.8	(4 marks, 5 min.)	[4, 5]

**COMPREHENSION (Q. NO. 1 TO 3)**

Let  $f(x)$  be a function such that it is thrice differentiable in  $(a, b)$ . Consider a function

$$\phi(x) = f(b) - f(x) - (b-x)f'(x) - \frac{(b-x)^2}{2} f''(x) - (b-x)^3 \lambda.$$

and  $\phi(x)$  follows all conditions of Rolle's theorem

on  $[a, b]$

- If there exist some number  $c \in (a, b)$  such that  $\phi'(c) = 0$  and  $f(b) = f(a) + (b-a)f'(a) + \frac{(b-a)^2}{2} f''(a) + \mu(b-a)^3 f'''(c)$ , then  $\mu$  is

(A)  $\frac{1}{2}$                       (B)  $\frac{1}{6}$                       (C)  $\frac{1}{8}$                       (D)  $-\frac{1}{2}$
- Let  $f(x) = x^4 - 6x^3 + 12x^2 - 8x + 3$ . If Rolle's theorem is applicable to  $\phi(x)$  on  $[2, 2+h]$  and there exist  $c \in (2, 2+h)$  such that  $\phi'(c) = 0$  and  $\frac{f(2+h) - f(2)}{h^3} = g(c)$ , then slope of tangent of curve  $y = g(x)$  at  $x = 5$  is

(A) 4                      (B) 5                      (C) 6                      (D) 10
- Let  $f(x) = e^{2x}$  and  $b = a + h$ . If there exists a real number  $\theta \in (0, 1)$  such that  $\phi(a + \theta h) = 0$  and  $\frac{e^{2h} - 1 - 2h - 2h^2}{h^3} = Ae^{B\theta h}$ , then the value of  $\frac{2B}{A}$  is equal to

(A) 4                      (B) 3                      (C) 6                      (D) 8
- The curve  $y = x^3 + x^2 - x$  has two horizontal tangents. The distance between these two horizontal lines, is

(A)  $\frac{13}{9}$                       (B)  $\frac{11}{9}$                       (C)  $\frac{22}{27}$                       (D)  $\frac{32}{27}$

5. If  $a, b > 0$ , then minimum value of  $y = \frac{b^2}{a-x} + \frac{a^2}{x}$  in  $(0, a)$  is
- (A)  $\frac{a+b}{a}$                       (B)  $\frac{ab}{a+b}$                       (C)  $\frac{1}{a} + \frac{1}{b}$                       (D) none of these
6. Find maximum possible area that can be enclosed by a wire of length 20 cm by bending it in form of a circular sector.
- (A) 10                      (B) 25                      (C) 30                      (D) 20
7. If the sides  $a, b, c$  of a triangle ABC are the roots of the equation  $x^3 - 13x^2 + 54x - 72 = 0$ , then the value of  $\frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c}$  is equal to (with usual notation in  $\triangle ABC$ )
- (A)  $\frac{169}{144}$                       (B)  $\frac{61}{72}$                       (C)  $\frac{61}{144}$                       (D)  $\frac{169}{72}$
8. If  $x = e^t \sin t, y = e^t \cos t$ , show that  $\frac{d^2y}{dx^2} = \frac{-2(x^2 + y^2)}{(x+y)^3}$

## Answers Key

1. (B)                      2. (A)                      3. (B)                      4. (D)
5. (D)                      6. (B)                      7. (C)

